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Effects of Self-Induction in an Iron Cylinder When Traversed by Alternating Currents

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7. Nitric oxide greatly accelerates the decomposition.

8. The rate of decomposition is a linear function of the oxygen pressure. A greater effect is produced by diluting with nitrogen than by simply reducing the pressure of the oxygen.

9. At 100° the reaction appears to be very slightly reversible.

10. Finally, the decomposition appears to take place mainly (if not entirely) at the surfaces with which the ozone is in contact, and pressure measurements give no indication of the number of molecules reacting.

The expense incurred in this investigation has been defrayed by a grant from the Royal Society.

Effects of Self-induction in an Iron Cylinder when traversed by Alternating Currents.[*]

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In a previous paper[†] the effects of self-induction in an iron cylinder were studied when a continuous current flowing through the cylinder in a direction parallel with its axis of figure was suddenly reversed and again maintained steady. In the experiments reported in the present paper the currents in the cylinder were made to alternate in the following manner. A continuous current dynamo, capable of giving currents up to 2000 amperes or more, was weakly excited and its brushes were short-circuited by the cylinder to be experimented upon in series with the shunt of a moving-coil ampere-meter. The brushes were moved round the commutator by aid of a worm and worm-wheel from the position of maximum to that of zero current. The field was then reversed and the brushes moved back to their initial position.

By continuing these operations an alternating current was caused to flow through the cylinder, and its periodic time was controlled by the speed at which the brushes were moved. The worm axle was uniformly rotated by hand at a speed determined by the operator listening to a seconds clock.

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[*] In connection with this research, I wish to acknowledge a grant voted to me by the Council of the Royal Society out of the Government Grant Fund.

[†] 'Roy. Soc. Proc.,' A, vol. 78, p. 22, 1906.

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Simultaneously readings were taken at known epochs on (*a*) each of three dead-beat galvanometers connected to exploring coils threaded through holes in the mass of the cylinder for the purpose of obtaining the E.M.F.'s at different depths due to the rate of change of the magnetic induction, and on (*b*) the moving-coil ampere-meter.

The cylinder is of mild steel and has a diameter and length each equal to 10 inches (25·4 cm.). It is provided with holes drilled in a plane containing its axis of figure in such manner that exploring coils could be threaded to enclose certain portions of that plane. The coils are referred to respectively as 1, 2, 3, and are each 2 inches wide in a direction parallel with the axis of figure and mid-way between the ends of the cylinder. Their depths in a radial direction are respectively 1, 2, and 2 inches and therefore their average radii are 0·5, 2, and 4 inches respectively. The current was passed into the cylinder by aid of the massive gunmetal castings used in the experiments previously described.* The deflections of the galvanometers have been reduced to volts per turn per square centimetre and are plotted in the case of two of the experiments in figs. 1 and 2. The curves are numbered 1, 2, 3 to correspond with the particular coils from which they were obtained. Time progresses from left to right of each figure.

Magnetic Induction (*B*) *as affected by Variation of Periodic Time and Total Current.*—On integrating the E.M.F. curves the magnetic induction (B) has been obtained. In Table I, where the results are summarised, the maximum average values of the magnetic induction are set forth. Looking at the values of the maximum induction for periodic time 0·75 minute, and maximum total currents of 1000 and 2000 amperes, we notice how great is the shielding effect in the former case as compared with the latter. This effect is intimately connected with the average permeability of the iron, which, as will be shown, is higher with a maximum total current of 1000 amperes. A comparison of the maximum values of the magnetic induction for periodic time 0·75 minute and maximum total current of 500 and 1000 amperes shows that in those two cases the innermost coil is equally affected. For a maximum total current of 1000 amperes the percentage diminution of maximum B for coil 1 as the periodic time is varied from 6 to 0·75 minute is much greater than in the case of maximum total currents of 500 and 2000 amperes. It should be noted that in Table I the total amperes are given approximately, their accurate values are given in Table II.

Wave-form and Phase Displacement as affected by Variation of Periodic Time and Total Current.—Figs. 1 and 2 are given as showing two extreme cases. In each diagram c_1, c_2, c_3 are the currents interior to the radii $\frac{1}{2}$, 2, and 4 inches respectively. In fig. 1 the periodic time is 6 minutes and the total current 1000 amperes, and the shielding effect is comparatively small. In fig. 2 the periodic time is 0·75 minute and the total current 1000 amperes, and this

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* See 'Roy. Soc. Proc.,' vol. 69, p. 440, fig. 1.

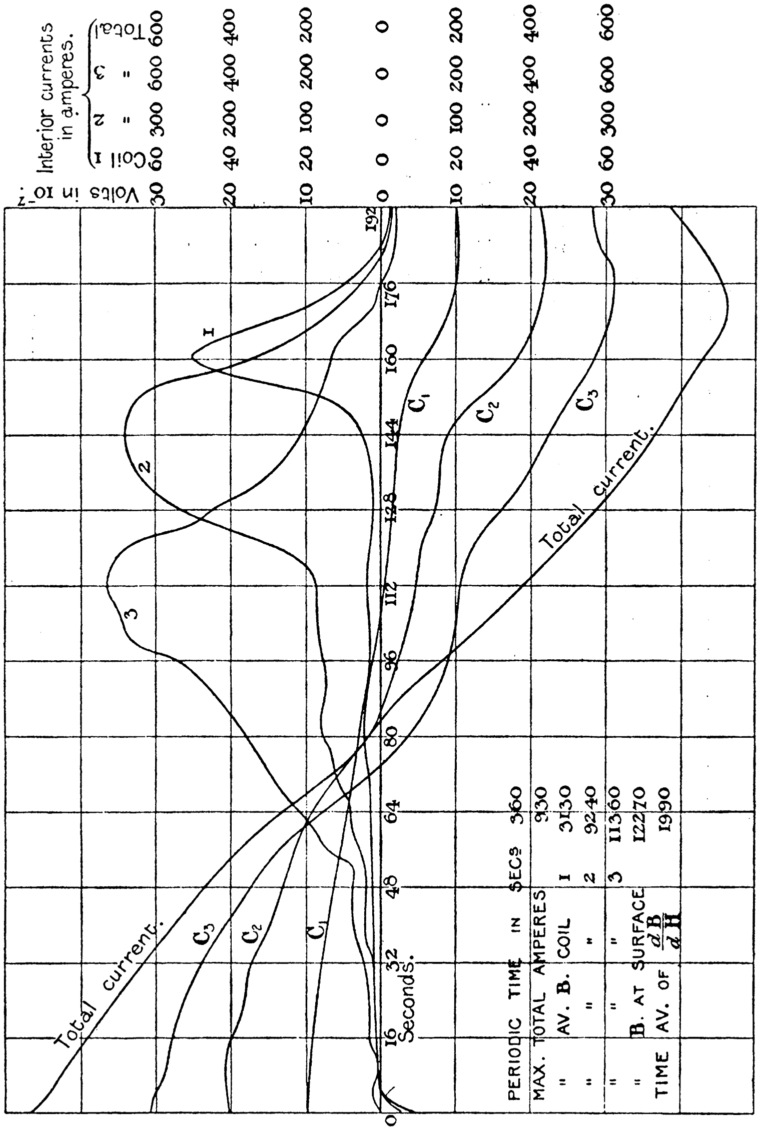

Fig. 1.

is one case in which the centre of the cylinder is practically devoid of current. It may be generally remarked that, referring the phase displacement of the E.M.F. curves to the maximum of current for a given frequency, the maximum E.M.F. occurs earlier as the current is increased. For a given total current

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the maximum of E.M.F. occurs later as the frequency is increased. The wave-forms of the E.M.F.'s are very much more peaked for the higher currents and the longer periodic times. The peaks of the E.M.F. curves cease to be so prominent at the high frequencies, but they are retained more with the large total currents than with the small ones.

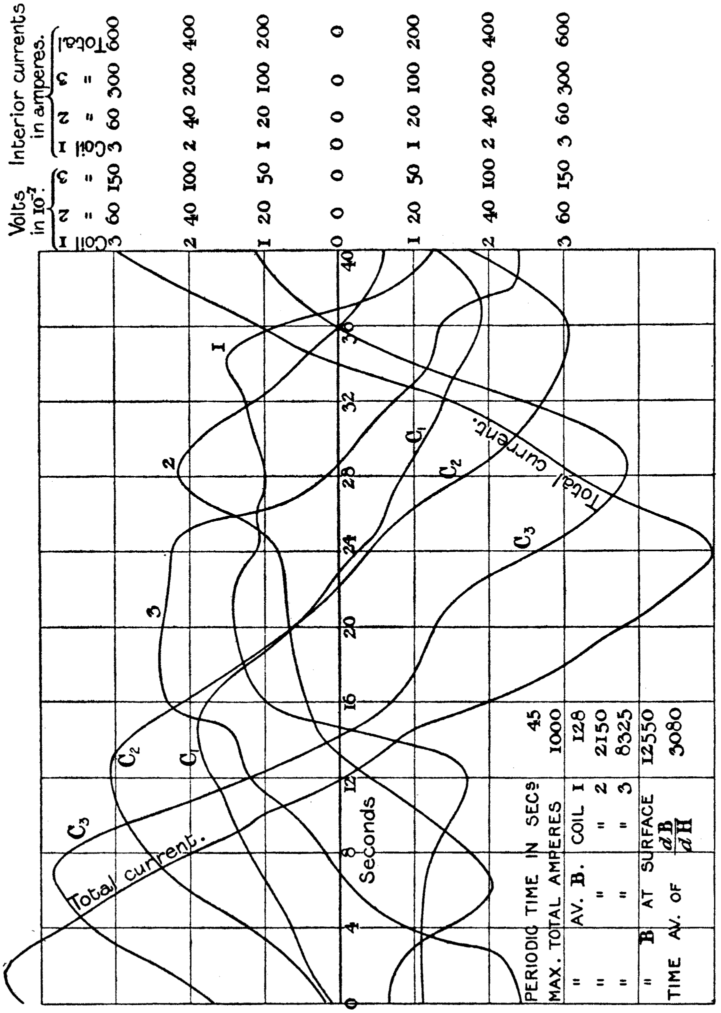

FIG. 2.

Energy dissipated by Electric Current in overcoming Resistance and by Magnetic Hysteresis.—For the varying conditions of frequency and total current the C^2R watts per centimetre length of the cylinder have been calculated and compared with the C^2R watts which would be found, if a continuous current equal in value to the root-mean-square value of the

Table I.—Maximum Average Induction B and Average Permeability.

Maximum total amperes.*	Periodic time in minutes. Frequency	0·75. 1/45.	1·5. 1/90.	3. 1/180.	6. 1/360.
2000	B Coil 1	3,460	4,440	4,720	8,960
	B Coil 2	6,050	12,000	12,640	12,040
	B Coil 3	12,600	14,080	14,160	13,560
	B at surface of cylinder	14,240	14,380	14,380	14,340
	Permeability	2,520	2,800	2,140	1,260
1500	B Coil 1	206			
	B Coil 2	4,460			
	B Coil 3	8,940			
	B at surface of cylinder	13,780			
	Permeability	3,050			
1000	B Coil 1	128	680	2,040	3,130
	B Coil 2	2,150 †	6,200	8,200	9,240 ‡
	B Coil 3	8,325	10,620	11,100	11,360
	B at surface of cylinder	12,550	12,480	12,150	12,270
	Permeability	3,080	3,060	2,690	1,990
500	B Coil 1	128	215	294	576
	B Coil 2	880	2,070	3,240	4,736
	B Coil 3	4,580	5,500	6,620	6,940
	B at surface of cylinder	8,400	7,300	7,250	7,000
	Permeability	2,010	2,260	2,310	2,130

* These are approximate values. For accurate values, see Table II.

† Fig. 2. ‡ Fig. 1.

alternating current were allowed to traverse the cylinder: (*a*) on the assumption of constant current density; (*b*) having regard to the variation of current density which was found actually to exist, owing to end effect. An attempt is made to compare the ratios obtained with the average permeability of the material under the conditions of test. The watts due to magnetic hysteresis have also been calculated, and a comparison made between them and the watts which would be found if the distribution of magnetic induction under continuous current were assumed to persist at the particular frequency. By a different application of the same principles, an attempt has been made to check the values of the total watts due to C^2R and magnetic hysteresis found separately. Finally, the ratios of the watts due to C^2R and to magnetic hysteresis are given. The results are set forth in Table II.

(1) The C^2R watts have been calculated as follows. The E.M.F. curves previously integrated have been used to give the average value of the magnetic induction (B) over the three annuli at any instant of time. From the hysteresis loops for the material, obtained by the ballistic galvanometer, the corresponding values of H have been obtained. In finding the currents from

Table II.

Frequency.	Total current in amperes.		C^2R loss per cm. length of cylinder in watts.					Average permeability.	Watts lost by magnetic hysteresis per cm. length, in 10^{-5}.	Total watts per cm. length, in 10^{-5}.	Ratio C^2R/hysteresis = A/D.
	Max. value.	R.M.S. value.	Alternate current, in 10^{-5}.	Continuous current, uniform distribution, in 10^{-5}.	Ratio A/B.	Continuous current, actual distribution, in 10^{-5}.	Ratio A/C.				
			A.	B.		C.			D.		
1/45	1860	1200	4700	3390	1·39	3790	1·24	2520	824	5520	5·74
(0·0222)	1520	915	2970	1985	1·50	2220	1·34	3050	663	3630	4·48
	1000*	598	1490	847	1·76	947	1·57	3080	381	1870	3·90
	500	299	325	212	1·53	237	1·37	2010	169	494	1·92
1/90	2000	1270	4770	3830	1·25	4280	1·12	2800	540	5310	8·85
(0·0111)	980	627	1230	930	1·32	1040	1·18	3060	298	1530	4·12
	430	269	238	171	1·39	191	1·24	2260	97	335	2·46
1/180	2000	1260	4680	3730	1·25	4160	1·12	2140	286	4960	16·4
(0·00556)	900	588	1030	819	1·25	915	1·12	2690	175	1200	5·88
	435	267	226	169	1·34	189	1·20	2310	69	295	3·28
1/360	1950	1270	4290	3810	1·13	4250	1·01	1260	140	4430	30·6
(0·00278)	930†	591	987	831	1·18	929	1·06	1990	94	1080	10·5
	420	269	212	172	1·23	192	1·10	2130	36	248	5·89

* Fig. 2. † Fig. 1.

these values of H, it is assumed that the average value of B in each annulus occurs at the average radius of the annulus. The error thus introduced is not very serious. The total interior currents thus found have been plotted for each instant of time in terms of the radius of the cylinder. From these curves the current over each of five annuli, each having 1 inch radial depth, has been estimated. Its squared value has been multiplied by the resistance of the annulus, and hence the total C^2R watts at any instant of time have been found. The time-average of the total values has then been taken over a half period of the alternating current. Referring to Table II, it will be seen that at frequency 1/45 (0·0222) the ratio of the watts with alternating to those with continuous current rises to a maximum for a total current of 1000 amperes. In attempting to connect these results with the average permeability of the material, the latter has been found as follows.*

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* The time-average of dB/dH has been chosen in preference to the time-average of B/H, because the latter takes no account of the previous history of the material, which is vital when the effects of magnetic hysteresis have to be considered. For example, when the iron is subjected to a periodic magnetising force of fixed direction relative to the iron,

At equal intervals of time the values of the magnetic induction (B) have been taken, and from the hysteresis loops the ratio dB/dH has been found. The time average of dB/dH per coil was then multiplied by the volume of the annulus, and the sum of the averages of the three annuli was divided by the total volume of the cylinder. It will be seen that at frequency 1/45 the maximum value of this average permeability also occurs with a total current of 1000 amperes. Unfortunately, the ratio of the watts with alternating to those with continuous current at the other frequencies in Table II is not sufficiently large to justify an accurate comparison. All through it is striking how small is the variation of the average permeability. Its value is greatly affected by variation of wave-form. For instance, in fig. 1, where the change of magnetic induction B is rapidly made, the average permeability depends largely upon the maximum value of B. In fig. 2 the change of magnetic induction B gradually takes place over the half period, in which case the average permeability is more dependent upon the intermediate values of B.

(2) The watts due to magnetic hysteresis have been calculated from the area of the hysteresis loops as follows. The maximum values of the magnetic induction (B) over each of the five annuli, corresponding to the maximum values of H obtained from the distribution curves of current have been found. The ergs per cycle per cubic centimetre given by the hysteresis loop corresponding to the maximum value of B have been multiplied by the frequency and the volume of the annulus and by 10^{-7} to reduce to watts. These values have then been added, and the sums are given in Table II. Referring to the experiment at frequency 1/45, in which the hysteresis watts are 824, the watts which would be dissipated if the distribution under continuous current (rendered non-uniform through end effect) persisted at this frequency are 1170, and if the distribution under continuous current of constant density obtained the watts would have been 1040. This bears out the statement that the watts due to hysteresis for a given total current are diminished by the effect of internal self-induction, commonly referred to as "skin effect."

(3) Using the equation $E = Rx + \frac{dI}{dt}$, where E is the impressed potential difference, R is the ohmic resistance, x the current, and I the total magnetic field external to the annulus considered, it was thought possible to check the values above found by a different process. The value of dI/dt was found

the ratio B/H becomes infinitely great when $H = 0$; whereas the magnetic induction (B) at that moment depends entirely upon the previously applied magnetising forces. This *is* taken account of in the differential dB/dH, as this is the slope of the curve at the point.

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between the surface of the cylinder and the radii 1, 2, 3, and 4 inches respectively, as follows. From the distribution curves of current, and the hysteresis loops, the wave-forms of B were plotted for average radii $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, and $4\frac{1}{2}$ inches. These were then differentiated to find E.M.F.'s, which were multiplied by the area of the annulus considered, and the values for the respective annuli were added together; this gave the value dI/dt. The effect of the magnetic field in the surrounding air space was proved to be negligible. The watts taken by five annuli each $\frac{1}{4}$ inch deep and having average radii of 1, 2, 3, 4, and $4\frac{7}{8}$ inches were found as follows. From the distribution curves the current over each annulus was obtained and thence C^2R; also the product of the E.M.F. (dI/dt) external to each annulus and the current (x) was found. The sum of these gave at any instant the total watts. The average over a period then followed. Dividing this average by the volume of the annulus the watts per cubic centimetre were obtained at its average radius. A distribution curve was then plotted giving the watts per cubic centimetre in terms of radius, and from this the average watts at radii $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, and $4\frac{1}{2}$ inches were obtained. Multiplying these values respectively by the volume of the annulus and adding them together, the total watts were obtained. The hysteresis loss $x dI/dt$ could also be found separately in a similar manner. Three cases have been worked out, and the results are given in Table III, together with the corresponding figures from Table II. This latter method could not be expected to yield such accurate results so far as hysteresis is concerned, since it depends upon the product of an E.M.F. and current largely displaced in phase.

Table III.

Frequency.	Total maximum current in amperes.	Total watts taken from Table II.	Total watts obtained by alternative method.
1/45 (0·0222)	1860	5520	6110
1/360 (0·00278)	1950	4430	4460
1/45 (0·0222)	500	494	480

(4) Referring to the ratio of the C^2R and hysteresis watts, it is interesting to note that for a given total current the ratio diminishes as the frequency increases, the increase in the hysteresis watts due to increased frequency much more than counterbalances the decrease, owing to the greater skin

effect. At frequency 1/45 (0·0222) and maximum total current of 1000 amperes the skin effect is, however, so great (see Table I) as to give a ratio 3·9, not much less than that given by practically the same total current at half the frequency.

Experiments at High Frequency.—The cylinder was transferred to the secondary circuit of a transformer, and various currents up to 1000 amperes were passed through it at a frequency of about 11 periods per second. A dead-beat galvanometer was employed in conjunction with a commutator driven by spur gearing at a speed differing from that of the alternator by 1 part in 10,000, for the purpose of giving the integrals of the curves of E.M.F. of the exploring coils 1, 2, 3. Large E.M.F.'s were, of course, obtained in the case of coil 3, but none in the case of coils 2 and 1. The sensibility of the galvanometer was such that a terminal voltage of 7×10^{-6} could be detected. Table IV summarises the results obtained. It must be borne in mind that, as in the foregoing experiments, the average magnetic induction B is assumed to be effective at the average radius of the coil. On this assumption the total current interior to the radius 4 inches is only from 3 to 5 per cent. of the total current in the cylinder. The actual skin effect is much greater than these figures indicate.

Table IV.

Frequency.	Total current in amperes.		Maximum induction B at surface.	Maximum of average induction B through Coil 3.	Current in amperes interior to radius 4 ins. (maximum value).
	R.M.S.	Maximum.			
11·66	987	1800	14,250	625	56·4
10·0	640	1090	12,800	490	47·2
10·93	371	691	10,200	268	30·3

Application of Results to other Sections.—Comparing two cylinders of the same material whose diameters are as $1:n$, it was shown* that, provided the total currents are as $1:n$ and the frequencies as $n^2:1$, the magnetic forces at similar radii will be the same. It follows also that the total watts due to C^2R and magnetic hysteresis will be the same. Thus, for example, the watts given in Table II refer also to a wire 1 cm. long and 0·1 inch diameter, provided the respective frequencies are 222, 111, 55, and 28, for each of which the total amperes are 5, 10, and 20. The curves in figs. 1 and 2 can also refer to a cylinder 0·1 inch diameter, provided the respective frequencies are 28 and 222, and the total current is 10 amperes, having the same wave-form. Cylinders having other diameters could be dealt with in the same way.

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* *Ibid.*

[*Note added February* 22, 1908.—Lord Rayleigh,* experimenting upon a hard Swedish iron wire 0·16 cm. diameter, found that the ratio of the resistance with alternating to that with continuous currents was about 1·2 when the frequency of the alternating currents was 1050. On the above assumptions this would correspond with a frequency of 0·04 in the case of the cylinder 10 inches in diameter. Table II shows that at frequency 0·022 and 299 R.M.S. amperes the ratio of the resistances is 1·37. Having regard to the fact that the specific resistances and the magnetic qualities of the materials of the two cylinders may have been different, and to the probability that the current in Lord Rayleigh's experiments was less than 1·9 amperes, which is the current corresponding to 299 amperes in the 10-inch cylinder (thereby giving rise to a smaller average permeability), the difference in the ratio of resistances corresponding to given frequencies may be accounted for.]

In conclusion, I wish to thank Mr. Alexander Siemens for the loan of the dynamo used in these experiments. I also wish to acknowledge the help I have received from Mr. A. E. O'dell and Mr. G. F. O'dell. Mr. Robertson, Mr. Franks, and some of my senior students gave me valuable assistance in the experimental part of the work, for which I wish to express my thanks.

* 'Phil. Mag.,' 1886.

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